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An Optimal Analysis for an n-gate Policy Queue System: a Cycle Period Distribution Approach

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Abstract

A reflection on the characteristics of the ATM traffic source models which are characterized by both burstness and a high correlation at the entry point when transmitting packet data has been central to studies in queueing theory in particular. The interest of this study is being tailored to the need for performance evaluation and optimization in certain network switches. This paper analyzes a simple data traffic system undergoing the typical cycle period distribution with a deterministic server and Markov Modulated Bernoulli Process (MMBP) at the arrival on a discrete time scale. With certain assumptions in place, the next busy period is considered given that it delays when the system is opened at the entry point.
The analysis is used to demonstrate the influence of the utility parameter ρ in an idle period distribution of an MMBP arrival flow.

Keywords: Diffusion, Bursty Traffic, Idle Period Distribution, Discrete Time, Ergodicity.

1. Introduction

The issue that surrounds the Asynchronous Transfer Mode (ATM) network has not barred its usefulness in Broadband-Integrated Services Digital Networks (B-ISDN) for wide range of bandwidth requirements, bit rate variations (Variable Bit Rate traffic) and certain other phenomena within the network connection. The packetized traffic network has kept a focus on the peak cell rate [3] and [6] with the fact that the ATM has built-in mechanisms that allows for provision an appreciable quality of service delivery at different times and on different traffic systems. However, one of the major challenges for ATM networks aside statistical multiplexing, has been the cycle period distribution. This distribution mainly rejuvenates the server and an ease in traffic flow within a network. Under certain conditions, the ATM traffic is unpredictable, it is bursty (e.g., data services), and has a high correlation at the arrival point. This causes unease in the system particularly when trying to model the system network with the model it worked with in a last study. Within the debate of ATM traffic characterization, several parameters have been introduced (e.g., peak/mean cell rate ratio, burst length, etc.) and following [6] in an attempt to get a much approximate quality of service guarantee, the Markov Modulated Process is convincingly appropriate in the discrete-time (Markov chain) sense (DTMC). We follow the approach of the idle period distribution; the time just before the arrival of the first customer or just after the departure of the last customer in the system. The organization of this paper is as follows: a quick review of literature in section 2; a frame of the model and its mechanism in section 2; what follows is a mathematical construct of the problem, results and discussion; asymptote of the process and; a conclusion and recommendation for section 3, 4 and 5.

2. Model Description

Let V_t be an IID sequence of random variable of the length of idle period distribution in an $n-$diffusion policy system whose arrival process is MMBP into a deterministic service and, upon arrival in a complete cycle period distribution system. This diffusion takes proportions of available jobs from an i^{th} to $(i+1)^{th}$ phase with a sharing formula commensurate the decreasing buffer size as available jobs complete the process

Preprint submitted to Performance Evaluation *June 13, 2017* in the system. This n system policy measures up to the time when a filter diffuses out the available job into the next buffer for an onward forward process in the direction towards exiting the system. With the system closed at the entry point until the next idle period, the server is busy for k time until the last job is served. On a fresh arrival, the time until the next idle period, the job passes into n servers until it exits the system where it needs compete for the available space because the buffer is smaller as n goes large, the job at c_i is an implication of the fact that it was passed from c_{i-1}, $i \geq 2$ whereas it is either empty or has a residual portion of the leftover from the last dispatch which is a flow in from c_{i-2}. If c_{i+2} is idle in a busy system just before an arrival into it, no sooner is the buffer is occupied if the flow is yet to get to c_{i+} and hence the idle period is of length $\rho \times V_{n-(i+2)}$ upon awaiting the flow of the approaching job (ρ is the system utility measured as the ratio of both rates of arrival and service response in the system). Thus, we see that the next idle period is defined for $k \geq n$ for the spread and diffusion through the system channels.

Hence, we consider the system at departure instants and its matrix probability into transition which are given as;

$$P = \begin{array}{c c} & \begin{array}{cc} ON & OFF \end{array} \\ \begin{array}{c} ON \\ OFF \end{array} & \begin{bmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{bmatrix} \end{array} \tag{1}$$

where

a= Pr{an arrival at time t—an arrival at t-1}

$1-a$ = Pr{an arrival at time t—no arrival at t-1}

b = Pr{no arrival at time t—no arrival at t-1} and,

The steady state arrival flow into the system is

$$\Pi_1 = \frac{1-a}{2-a-b}, \quad \Pi_2 = \frac{1-b}{2-a-b} \tag{2}$$

$$P^{\infty} = \lim_{n\to\infty} P^n$$

With section 2 in place, let $X(t)$ be an i.i.d. random variable connoting a discrete time counting process of an idle period at service point (general) given arrival is Markov Modulated Bernoulli Process (MMBP) with $k-$phase diffusion, we express the probability in (2) as

$$P = \begin{bmatrix} a & (1-a)v + (1-a)(1-v) \\ 1-b & b \end{bmatrix} \tag{3}$$

v is the probability that at a certain instant t, the server is idle and there is no approaching customer. Under the limiting distribution condition, we have that

$$\pi_2 = \frac{1-a}{2-a-b} \tag{4}$$

becoming

$$\pi_2 = \frac{1-a}{2-a-b} v_t + \frac{1-a}{2-a-b}(1-v_t) \tag{5}$$

it follows that (5) gives

$$\pi_2 = \frac{1-a}{2-a-b} \times 1$$

Suppose v_t and $1-v_t$ are not symmetrical in a sense, hence, they are asymptotic. For a large convergence into steady state, it possesses a discrete jump process V_t, for large t, the quantity V_t approaches 1. Now with attention on V_t, it follows that

$$\pi_2 = \frac{1-a}{2-a-b} \{V_t\} \tag{6}$$

$$\pi_2 = \frac{1-a}{2-a-b} \{V_{t+0}, V_{t+1}, V_{t+2}, \ldots V_{t+k}\} \tag{7}$$

where $u = t + k$ and $k \in$
� $^{+}$

$$\pi_2 = \frac{1-a}{2-a-b}\{V_0, V_1, V_2, \ldots V_u\} \quad (8)$$

but $\{V_t\}$ is a growth function, we define that

$$V_t = e^{(1-\varphi)t} \quad (9)$$

$$\beta(s) = \quad e^{-st} dV_t \quad (10)$$

$$V_t^* = B^*[1-(s-\varphi)] \quad (11)$$

With the consideration of an arrival epoch, just after the completion of an idle period, it follows from (6) that

$$\pi_2 = \frac{1-a}{2-a-b}\{(1-V_0), (1-V_1), (1-V_2), \ldots (1-V_u)\} \quad (12)$$

$$V_t^{**} = \quad e^{-st} dV_t \qquad t = 1, 2, \ldots u \quad (13)$$

Let $V^*(s)$ be the LST of the DF of the length of I of an idle period. Since φ is a part function of the service time distribution, it is defined when the last customer has left the system.
Let B be the discrete count of the number just before the first arrival goes into service. We then have that,

$$V = y + V^1 + V^2 + V^3 + \ldots + V^B \quad (14)$$

Each B are mutually independent RV and of the same distribution

$$E[e^{-sy}] = E[e^{-s(y+V^1+V^2+V^3+\ldots+V^B)}] \quad (15)$$

$$E[e^{-sy}] = e^{-sy}E[e^{-sV^1}]E[e^{-sV^2}]E[e^{-sV^3}]\ldots E[e^{-sV^B}] \quad (16)$$

$$E[e^{-sy}] = e^{-sy}[V^*(s)]^k \quad (17)$$

Unconditioning on y, we have

$$E[e^{-sy}] = e^{-sy}[V^*(s)]^k\,[T] \times \Pr\{noarrival\} \quad (18)$$

where $T =$ sojourn time of the last customer at the end of a busy period, which kick starts the idle period.

$$E[e^{-sy}] = e^{-sy}[V^*(s)]^k\,[W_I \times B^*(s)]\ \Pr\{noarrival\} \quad (19)$$

where W_I is the waiting time of the last customer

$$E[e^{-sy}] = e^{-sy}[V^*(s)]^k\, T^{(\frac{1}{k})k}(s) \times \frac{1-a}{2-a-b} \times e^{-\varphi y} \quad (20)$$

With the assumption that the system is stable, we have that

$$Pr\{No-arrivals\} = \left(\frac{1-a}{2-a-b}\right)^x\left(\frac{1-b}{2-a-b}\right)^{1-x} \quad (21)$$

for $x = 1$.

$$E[e^{-sy}] = e^{-sy}[V^*(s)T^{\frac{1}{k}}(s)]^k \times \frac{1-a}{2-a-b} \times e^{-\varphi y} \quad (22)$$

after much simplification, (20) becomes

$$E[e^{-sy}] = e^{-[s+\varphi-T\dot{V}^*(s)]y} \quad (23)$$

for values of k, we have

$$\int_0^{\infty} E[e^{-sy}]dy = I^*[s+\varphi-\varphi(TV^*(s))^*] \quad (24)$$

3

Table 1: Waiting time for varing φ at different stages

stage	φ=0.1	φ=0.2	φ=0.3	φ=0.4	φ=0.5	φ=0.6	φ=0.7	φ=0.8	φ=0.9	φ=1.0
1	0.095	0.140	0.185	0.230	0.275	0.320	0.365	0.410	0.455	0.500
2	0.140	0.180	0.220	0.260	0.300	0.340	0.380	0.420	0.460	0.500
3	0.185	0.220	0.255	0.290	0.325	0.360	0.395	0.430	0.465	0.500
4	0.230	0.260	0.290	0.320	0.350	0.380	0.410	0.440	0.470	0.500
5	0.275	0.300	0.325	0.350	0.375	0.400	0.425	0.450	0.475	0.500
6	0.320	0.340	0.360	0.380	0.400	0.420	0.440	0.460	0.480	0.500
7	0.365	0.380	0.395	0.410	0.425	0.440	0.455	0.470	0.485	0.500
8	0.410	0.420	0.430	0.440	0.450	0.460	0.470	0.480	0.490	0.500
9	0.455	0.460	0.465	0.470	0.475	0.480	0.485	0.490	0.495	0.500
10	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

3. Numerical and Analytic Results with Discussion

Response time is the time it takes for the server to respond to n number of customers in N time interval.

1. The value W shows stability as z goes to zero, and it is turbulent as it goes further from zero.
2. For a constant s, the plot is asymptotically about z and W.
3. There is a sharp spike between $s \in (0, 0.25)$ signifying longer waits at z near its peak. This avalanche is conspicuous as ρ tends large.
4. The relationship between W and s tends to a negative gradient as s is increased.
5. The value W shows a sign of stability with small s with a constant z and a negative slope with a constant s.
6. Overall, the plot seems disjoint as the utility factor tends to 1; this is exhibited from the avalanche which is a result of the system being undefined at certain points.

since φ is a function of $T^*V^*(s)$ and our model works in the similitude of the M/G/1 queue system, we know from literature that

$$T = B^*(s)\frac{s(1-\rho)}{s-\lambda-\lambda B^*(s)} \tag{25}$$

where $0 < \rho < 1$. See [5].

(25) is defined from (22) to (24), hence we ascertain that the introduction of the HOV intervention like congestion control, increases the traffic prediction when accompanied by the plausibility of impediment to the free flow traffic and an enhancement to the priority lane.

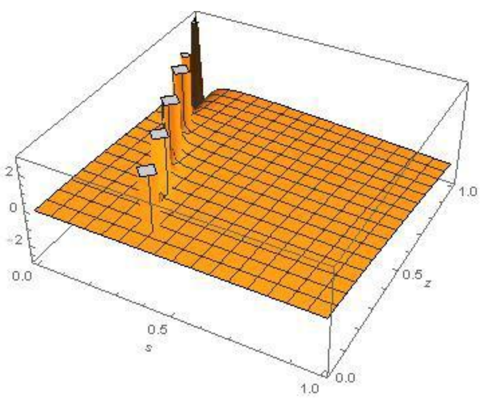

Figure 1: Idle period with $\rho = 0.10$

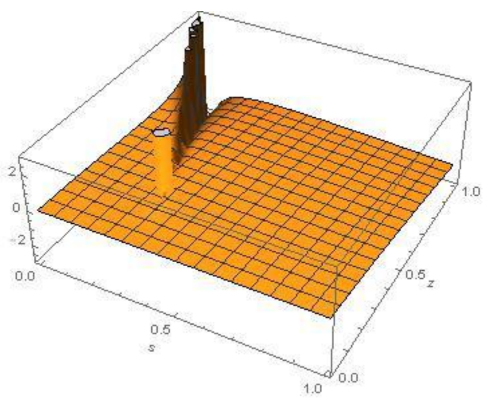

Figure 2: Idle period with $\rho = 0.25$

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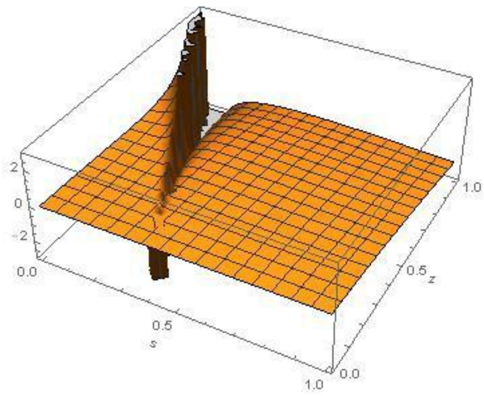

Figure 3: Idle period with $\rho = 0.50$

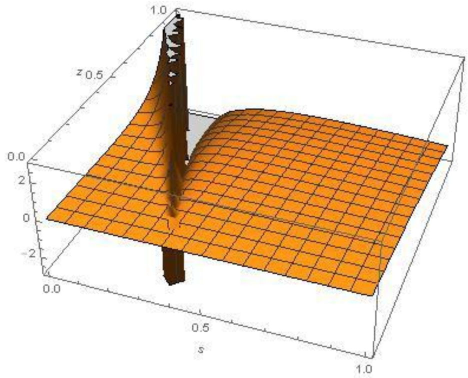

Figure 4: Idle period with $\rho = 0.75$

Table 2: Euclidean distance between φ of different stages

stage	1	2	3	4	5	6	7	8	9	10
Eulidean Distance	0.7597	0.6753	0.5909	0.5065	0.4220	0.3376	2532	0.1688	0.0844	0.00

Clearly, Table 1 is symmetrical and the values for φ =0.1 progressively attains that for the first stage at varying φs across column. It is not strange that for a shorter span, the range of waiting time for the process to begin is asymptotic about zero.

0.7597 is the Euclidean distance for the stage one in Table 1 and as we can see, the width as the stages progresses shrinks until 0.00.

4. Ergodicity of the Process

Recall that in the n bureaucracy in our system, a packet spends a unit time slot moving from bureaucracy i to $i + 1$. Upon an arbitrary time t it would spend $(n - t)$ time slot in the system, given that the deterministic service time of one time slot. Let the queueing system occupancy be defined by $N(t) = N_0(t), N_1(t), N_2(t), ..., N_n(t)$, given that each bureaucracy has equal and fixed buffer size. We introduce independent random variables $\xi(t)$ and $\eta(t)$, $t = 0, 1....$ assuming the values 0 and 1, by means of equalities [8]

$$Pr(\xi(t) = 1) = p_1, \quad Pr(\eta(t) = 1) = p_2, \qquad t = 0, 1, 2, ... \tag{26}$$

In this system, one server and the $(i - 1)^{th}$ input flow interact only with the $i^t h$ queue at each step. At the t^{th} step, a request from the input independently arrives at the first queue with probability p_1 and if the server contains a request at the t^{th} step, then it independently completes service with probability p_2 and, instead of the processed request, accesses the first queue for a new request. If the server is idle at the t^{th} step, then it accesses the first queue for a new request with probability 1 rather than p_2.
$\xi_i(t) = 1$, $\eta_i(t)$ can only be p given that the server at $i + 1$ is rendering a service. Thus for some $c_i = 0$, the capacity of the system at an arbitrary phase would bypass the queue capacity c_n. Though,

$$c_{n+1}(t) = c_{n+2}(t) = c_{n+3}(t) = ... = c_{n+m}(t) = 0, \qquad t \geq 0$$

Without loss of generality, based on the system, we assume that the random sequence $\xi(t)$ and $\eta(t)$, $t = 0, 1....$ are controlled by the arrival of requests in the system. By the virtue of this base knowledge, we can say that the spread is uniformly distributed and, thus

$$c_1(t) \geq c_2(t) \geq c_3(t) \geq ... \geq c_n(t), \qquad t \geq 0$$

For a more general time frame $t_1 < t_2 < t_3 < ... < t_n$, we define the event $A(t_1) = Pr\{\xi(t_1) = 1, \eta(t_1) = 1\}, A(t_2) = Pr\{\xi(t_2) = 1, \eta(t_2) = 1\}, ...$ Then, we can say that from (26)

$$_{M}A_{0,t} \geq \frac{A(t_{i+1}) - A(t_i)}{p_1 p_2} \geq 1 + \ _{A} \tag{27}$$

hence follows

$$1 - \frac{A(t_{i+1}) - A(t_i)}{p_1 p_2} \leq_M A_{0,t} \leq \ _{A} \tag{28}$$

In similar manner, the flow proportion is given as

$$\frac{N(t_{t+1}) - N(t_t)}{p_1 p_2} - \overline{N_{P-K}} < \ _{N} \tag{29}$$

where $\overline{N_{P-K}}$ is the average size in a specific queue from Pollaczek Khinchine mean-value theorem, and, $_{M}A_{0,t}$ is the moment of $A(t)$.

5. Conclusion

In this paper, the approach to the cycle period distribution for a Markov Modulated Bernoulli (arrival) Process MMBP is presented. This approach is a natural extension of a time-varying arrival event as applicable in a more general setting where an arrival of a packetized data is delayed via some bottleneck protocols at the input. With the system still maintaining its utility and defined functions, it is useful to find out what happens in the system at such times.
The methodology developed here is valid for a discrete-time queueing scheme and a Continuous-Time Markov Chain (CTMC), and on this base we make the performance comparison for the system utility where a and b are kept at 0.4 and 0.6 respectively $\rho = 0.1, 0.25, 0.50$ and 0.75.
As the mean service time increases, there is a slow decay in the idle period distribution, while the parameter of the Idle period distribution steeps and gradually flattens. The slope of the steep is smaller as ρ increases.

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